

Alternate Solution to Problem 2.

In order that $a^2 - 3a - 19 = 289m$ for some integers a, m , it is necessary that the discriminant $\Delta = 9 - 4(19 + 289m) = 85 + 34^2m$ be a perfect square. But since $17 \mid \Delta$ while $17^2 \nmid \Delta$, this is impossible. ■