

Solution to first part of the Problem 5

I keep the notations as in the official solution Fig.1. (so please refer to that Fig.1). Considering points in the co-ordinate plane

We have $T = m_1P + m_2Q$, $S = n_1A + n_2B$, where m_1, m_2, n_1, n_2 are positive reals such that $m_1 + m_2 = n_1 + n_2 = 1$,

$$\begin{aligned} \text{Now} \quad |S - T| &= |(n_1A + n_2B) - (m_1P + m_2Q)| \\ &= |(n_1A + n_2B)(m_1 + m_2) - (m_1P + m_2Q)(n_1 + n_2)| \\ &= |n_1m_1(A - P) + n_1m_2(A - Q) + n_2m_1(B - P) + n_2m_2(B - Q)| \\ &\leq n_1m_1|A - P| + n_1m_2|A - Q| + n_2m_1|B - P| + n_2m_2|B - Q| \\ &\leq n_1m_1 + n_1m_2 + n_2m_1 + n_2m_2 = 1 \end{aligned}$$