AN INTRODUCTION TO INEQUALITIES

Manjil P. Saikia[1]
Department of Mathematical Sciences,
Tezpur University, India

1. Properties

1. For any two real numbers a and b, we can have any one of the following three relations:
   (a) $a > b$
   (b) $a < b$
   (c) $a = b$
2. No square is negative!
$$a^2 \geq 0, \ \forall a \in \mathbb{R}.$$
3. A.M.-G.M. Inequality:For any two numbers a and b, we have $\frac{a+b}{2} \geq \sqrt{ab}$.

2. Problems

1. *Which of the fractions* $\frac{4567890123}{780123456}$ *and* $\frac{4567890124}{780123458}$ *is greater?*

2. *A man recieves* $\frac{p}{q}$*th part of Rs.a and* $\frac{q}{p}$*th part of Rs.a. He then gives away Rs.*$2a$*. Show that the man cannot lose in the transaction.*

3. *Determine which of the two numbers* 1000^{1000} *and* 1001^{999} *is greater?*

4. *Prove that* $3^{200} > 2^{300}$.

5. *Which is larger,* $9\sqrt{9!}$ *or* $10\sqrt{10!}$*?*

6. *If* p *and* q *are positive real numbers such that* $p + q = 1$*, then prove that*
$$(p + \frac{1}{p})^2 + (q + \frac{1}{q})^2 \geq \frac{25}{2}.$$

7. *If* a, b *and* c *are three real, then show that* $a^4 + b^4 + c^2 \geq 2\sqrt{2}abc$.

8. *Three positive numbers* a, b *and* c *satisfy* $a \geq b \leq c$ *and* $a + b + c \geq 1$*. Prove that* $a^2 + 3b^2 + 5c^2 \leq 1$.

9. *If* a, b *and* c *are three real numbers such that* $a^2 + b^2 + c^2 = 1$*, then prove that*
$$-\frac{1}{2} \leq ab + bc + ca \leq 1.$$

10. *Determine which of the following two numbers is bigger:*
$$\sqrt{5 + \sqrt{21}} + \sqrt{8 + \sqrt{55}} \text{ and } \sqrt{7 + \sqrt{33}} + \sqrt{9 + \sqrt{65}}.$$

[1]Email: manjil.saikia@gmail.com, manjil_msi09@agnee.tezu.ernet.in