

PROBLEMS IN ELEMENTARY NUMBER THEORY–CATEGORY II

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1. If the number 17293141519A is divisible by 33 then find out all possible values of A.
2. Find all pairs of integers (x, y) such that $x^2 - y^2 = 25$.
3. Without determining the square root say if 1729314159265352 is a square or not.
4. Show that any positive odd integer p is of the form $6q + 1$ or $6q + 3$ or $6q + 5$ where $q \in \mathbb{Z}$.
5. For what value of p is $2^{2p} + 7 \cdot 2^p + 1$ a perfect square?
6. Prove that none of the numbers of the sequence $11, 111, 1111, \dots, 111 \dots 111, \dots$ can be a perfect square.
7. If n is any natural number, prove that $n^2 + n - 1$ is always odd.
8. Find the smallest integer k which when divided by 6, 5, 4, 3 and 2 successively leaves remainder 5, 4, 3, 2 and 1 respectively.
9. Find the last digit of 7^{14} .
10. How many zeroes does $20!$ end in?
11. Prove that the product of four consecutive integers is always 1 less than a perfect square.
12. Prove that one of the integers $a, a + 2$ and $a + 4$ is divisible by 3.
13. Show that $3a^2 - 1$ is not a perfect square for any integer a .
14. Prove that $n^4 + 4$ is not a prime number $\forall n > 1$.
15. Prove that there is only one pair of non-zero integers whose sum is equal to their product.
16. Which four digit number $aabb$ is a square?
17. Find the four digit number which, on division by 131, yields a remainder of 112 and on division by 132 yields a remainder of 98.
18. A positive whole number $M < 100$ is represented in bases 2, 3 and 5 notation. It was found that in all the 3 cases the last digit is 1 while in exactly two out of the three cases the leading digit is 1. Find M .

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